ON THE DISTRIBUTION OF THE ENERGY IN THE SPECTRUM OF THE BLACK BODY AT LOW TEMPERATURES.[1]

By F. PASCHEN.

My observations on the energy spectra of different solid bodies[2] make it seem possible that the law derived by W. Wien[3] represents the emission of "the absolutely black body." In Wien's formula,

$$J = c_1 \lambda^{-5} e^{-\frac{c_2}{\lambda T}}, \tag{I}$$

where $J d\lambda$ is the energy between wave-lengths λ and $\lambda + d\lambda$ at the absolute temperature T, and c_1 and c_2 are constants, if we substitute λ^a for λ^{-5}, the value of a changing from body to body, my former observations are represented by the formulae. The value of a decreased from 6.4 to 5.2 in passing from reflecting platinum to strongly absorbing carbon.

The deviation of my former observations from the theoretical laws was such that that which was theoretically well-founded was not confirmed with certainty, while on the other hand that which was uncertain theoretically was rendered probable by the observations. The comprehensive formula of the law of emission is not supplied by theory in an unquestionable manner. If we assume the validity of the formula with the constant a derived from the experiments, then a must have the value 5, for Wien has proven that the intensity J_m of the maximum of energy varies as the fifth power of the temperature, which is the case in the assumed formula only when $a = 5$. In addition to this relation

[1] *Sitzungsberichte der Berliner Akademie ;* Session of the physical-mathematical section on April 27, 1899.

[2] *Wied. Ann.*, **58**, 455, 1896 ; **60**, 662, 1897. (The latter will be referred to as *loc. cit.*)

[3] *Wied. Ann.*, **58**, 662, 1896.

$$\frac{J_m}{T^5} = \text{constant}, \tag{1}$$

two further relations are firmly established by Wien, or follow from his accurate derivations, viz.:

$$\lambda_m \times T = \text{constant}, \tag{2}$$

or the wave-length of the maximum of the energy curve for temperature T is inversely proportional to the temperature; and (3) the relation that the ratio of the intensity J of wave-length λ to the intensity J_m of the energy maximum of wave-length λ_m, or $\frac{J}{J_m}$, is a function of $\frac{\lambda}{\lambda_m}$, and for the energy curves at different temperatures always the same function of $\frac{\lambda}{\lambda_m}$. In a logarithmic representation all the energy curves must be congruent.

I found laws (2) and (3) to be confirmed. The experiments gave for the last relation (3) the function

$$\frac{J}{J_m} = \left\{ \frac{\lambda_m}{\lambda} e^{\frac{\lambda - \lambda_m}{\lambda}} \right\}^a, \tag{4}$$

which theory left indeterminate. It was only on making certain doubtful assumptions that Wien succeeded in deducing formula I, and we may regard formula (4) with a set equal to 5 as the result of Wien's research.

We therefore have to investigate whether this function (4) holds good with the value of $a = 5$, if we approach closely to the radiation of the absolutely black body. This seemed probable from my experiments, since the function was valid for radiating bodies of very different absorptive power, and since a had already taken the value 5 for the blackest body. The confirmation of relation (1), which according to theory must hold first of all, will be a crucial test whether the arrangements of the experiments sufficiently comply with the postulates of the theory; for the constant c_2 of formula I can be determined from the measures with sufficient accuracy only when this relation — in my experience the most difficult to realize — also holds good.

In attempting to answer these questions I first determined the energy spectra of cavities, the sides of which were heated by baths. These experiments gave a somewhat closer approximation to law I than my former observations, as the relations 2, 3, and 4 appeared to be quite fully satisfied with $a = 5$, and as the observations were best represented by my previous formula with a value of about 5.2 for the exponent of λ; but it was impossible, however, to find the intensity of the maximum of energy exactly proportional to the fifth power of the temperature without overstepping the limits of error very considerably. The discrepancy might be due to the imperfect realization of the radiation of the black body, as the cavities had openings. When these were very much reduced, however, the emergent radiation still gave the discrepant result as before. The only thing in the experimental arrangements that could be held responsible for this was a variability of the absorptive power of the exposed bolometer strip with the wave-length. It therefore remained either to determine this or to so arrange the receiver of the radiation that it should constantly absorb the incident radiation as nearly completely as possible. I adopted the latter method and beg to show in what follows how far I have been able to approach to my aim. The experiments I communicate refer to the region of low temperatures and long wave-lengths. These experiments are the best adapted for judging of the blackening of the receiver, since my ordinary bolometers deviate most from my blackest ones just at this region.

ARRANGEMENT OF THE EXPERIMENTS.

The spectroscopic apparatus employed includes a fluor-spar prism loaned me by the firm of Carl Zeiss, which I had used before, and two silver concave mirrors of 35 cm focus, with precisely spherical figure, and so arranged that the astigmatism of the image was reduced as much as possible. The spectrum thus produced was so exceedingly sharp that the two broad absorption bands of aqueous vapor in the air of the room at $\lambda\,6.0\,\mu$ and $6.5\,\mu$ were resolved into numerous sharply defined bands, while between them at $\lambda\,6.26\,\mu$ was a place without appreciable absorption. The absorption band of carbon dioxide of the air of the room appeared narrow like a line, but at its deepest part extinguished more than two thirds of the original energy.

All of the bolometers with which the results below were obtained were simply strips of platinum of $\frac{1}{2000}$mm thickness and most had a breadth of 0.5mm, corresponding to an angle of 5 minutes in the spectrum. The piece upon which radiation fell had as accurate a rectangular form as possible. The slit-width was altered until the energy curve of a line (or of the image of the slit when the prism was removed) became as nearly as possible an isosceles triangle, as the correction for the impurity of the spectrum due to the width of the slit can be simply calculated for this case.[1] The exposed surface of the bolometer was blackened either with lampblack or according to the Lummer-Kurlbaum[2] method with platinum black. The layer of black was given two or three times the thickness prescribed in their rule or employed in my earlier bolometers. The extremely slight thickness of the bolometer strip gives the advantage that in spite of the thick covering with black the galvanometer deflection, even with a period of six seconds, behaves just as on breaking a shunt across the branch of the bolometer if the conductors to the sensitive parts are screened by metallic diaphragms, and protected above and below from the latter for a space of some 0.5 mm.

To produce a further effect of blackening, the bolometer strip was placed, according to the principle proposed by myself (*loc. cit.* p. 722) with its middle exactly at the center of a reflecting hollow shell which had a small aperture for the admission of the radiation. Only that hemisphere of the shell was present on which the radiation reflected from the strip could fall, while the strip was so fixed that it could reflect toward all possible parts of the hemisphere. The frame of the bolometer could be moved by a micrometer until the strip covered its image. For reflecting hemispheres I used one of 45 mm diameter with poor polish and an inaccurate surface. A second one, cut exactly spherical by

[1] C. RUNGE, *Schlömilch's Zeitschrift für Math. und Phys.*, **42**, 205, 1897.

[2] F. KURLBAUM, *Proceedings of the Physical Society of Berlin*, p. 11. June 14, 1895.

Zeiss of Jena, of diameter 50 mm, had a splendid polish.[1] Both were of German silver. For the present research with the bolometer strips from 5 to 7 mm long and comparatively broad, it was sufficient to project the spectrum sharp in the plane of the strip. If one is to work with higher dispersion it is better to throw a sharp image of the spectrum in the plane of the slit of the hemisphere, and to make this slit equal to the image of a line. A very perfect arrangement in respect to the blackening of the strip and the projection of the spectrum is obtained if a bolometer is placed in the central plane of the hemisphere, of such width that the incident radiation does not fully cover its strip in length and breadth, and so that the radiation diffusely reflected from the strip always returns upon sensitive parts in spite of the aberrations for rays out of the center of the sphere. This also has the advantage of allowing, even with high dispersions, the use of a sensitive surface bolometer, which of course must be opaque for its whole breadth. The visual observation of the spectrum projected upon the exterior wall of the hemisphere is effected by a properly adjusted mirror. Since the bolometer strip reflected upon itself is heated by the current, and gets a great part of its radiation back again, the equality of the resistances of the bolometer will be disturbed. In order that the second strip should also get back at least a part of its radiation, I attached a plane silver mirror parallel to the central plane at the position of its image. With the strongest currents permissible, however (0.05 amperes), no such inequality of resistances arose to render the bolometer useless, so that the mirror was unnecessary.

I used for galvanometer a newly constructed instrument which is more sensitive than my former one,[2] and is rendered so far astatic that its directing force is chiefly due to the quartz fiber. Only in this way was it possible to work with so delicate a galvanometer at a place strongly affected by the earth-currents from a near by electric street railway.

[1] Viewed with a microscope magnifying about 150 times, the image of a fine thread in the center appeared as sharp as the thread itself.

[2] *Wied. Ann.*, 50, 417, 1893.

The radiating cavities were cylindric or pear shaped and of such size that the distance was from 10 to 13 cm from the rear wall to the aperture of about 1 sq. cm area. Only a part of the rear wall could send radiations through the suitably diaphragmed aperture. A larger vessel enclosed the hollow radiator, and served either to boil a liquid or to contain the vapors led in from a special vessel in which the liquid was boiled. The hollow radiator was always in the vapors. The arrangement for different temperatures was briefly as follows:

1. 100° C. The steam from water boiling in a flask was led into the enveloping vessel. Hollow radiators of copper oxide, lampblack and platinum black were investigated.

2. 190° C. Impure commercial aniline was boiled with a return condenser in the metallic enveloping vessel. The temperature of the cavity was determined each time by a thermometer certified by the *Reichsanstalt.*

3. 304° C. Arrangement the same as last, with impure commercial di-phenylamin. Cavities covered with copper oxide or lampblack.

4. 450° C. Sulphur was boiled in the jacket of a double-walled glass vessel with a condensing tube. The inner vessel constituted the cavity. Its surface, which had been roughened by etching, was covered by a thick layer of copper oxide. In several experiments a lampblack layer was put on over this, but it disappeared very quickly.

The glass vessel was closely surrounded by the metallic jacket. The flame played freely only under the boiling sulphur. With this arrangement the temperature of the interior of the cavity was from 449° to 451° C., according to the atmospheric pressure and the arrangement of the flame, being indicated by a mercury thermometer extending to 550° C. (certified by the *Reichsanstalt*) or by a thermo-element referred to this.

Differences of temperature as small as 1° C. with aniline, and 3° C. with sulphur, were determined by a thermo-element in the interior of the cavity, in some cases. The temperature was then taken at the point of the rear wall opposite the aperture. All of the cavities could easily be provided with smaller apertures by the introduction of diaphragms. Some of the cavities in zinc were covered with platinum black by the reduction of a solution of 1 per cent. platinum chloride and 0.1 per cent. lead acetate on the zinc wall. The blackening thus obtained almost surpasses that of the electrolytic process of Lummer and Kurlbaum if the basic salts of zinc have been removed with dilute acetic acid. The cavities were placed in front of the spectroscope in such a way that the radiation from the aperture entirely filled a fixed diaphragm before the prism.

Energy curves only were observed, the sensitiveness of the bolometer in the different series being compared by the well-known shunt method of Ångström, or by comparative observations of a constant source of radiation, or by the measurement of the bolometer current. I proceeded along the spectrum to $\lambda\ 5\mu$ by steps of the width of the bolometer strip (an angle of about $5'$), since the correction for impurity of the spectrum can then be very easily and accurately computed, as shown by Runge.

The first of the strong absorptions of aqueous vapor begins at 5μ[1]. At 6.26μ there is a narrow place between the two bands which shows no absorption in my spectrum, but at which the correction for impurity of spectrum reaches a considerable amount on account of the falling off of energy on the two sides. Beyond the second strong water absorption, a region begins at about 7.7μ, which exhibits as far as about 9.3μ no appreciable absorption by the air of the room, but does show absorption by the substance of the prism. In this region the four wave-lengths, 7.738μ, 8.246μ, 8.806μ and 9.324μ were investigated, account being taken of the absorption by the prism.

For the calculation of the normal energy spectrum, in which the energy contained in a constant, narrow range of wave-lengths is considered a function of the wave-length, I made use of my new determination of the dispersion of fluor-spar, in which the

[1] PASCHEN, *Wied. Ann.*, 53, 335, 1894. Graphical representations of the energy curve covering these regions of absorption may be found in *Wied. Ann.*, 52, Plate, Fig. 1; 51, Plate, Fig. 2, Curve 1.

prism and grating spectra had considerably greater dispersion and sharpness than in my earlier determinations, so that the results had errors from two to three times smaller than before. My first measure[1] proved the more correct, and the accuracy of the determinations of the constants of Ketteler's dispersion formula was increased, so that the differential quotients of this formula which were used in the reduction have throughout the spectrum errors less than those of observation (at least in this research).

In addition to the corrections I have previously dealt with in detail, for width of the slit, for reflection from the surfaces of the prisms and mirror, and for the spectrum of the slide[2] in front of the slit serving as a zero point, the absorption of the prism was eliminated for the region from 7.7μ to 10μ (see below).

MEASUREMENTS.

I will cite only the result of one of the series of measures with an ordinary lampblacked bolometer, for which series the radiating cavities were arranged in the same way as for the later experiments. Other measurements with ordinary bolometers and cavities heated by baths gave similar results.

BOLOMETER STRIP WITH A THICK LAYER OF LAMPBLACK OF AN ANGULAR WIDTH IN THE SPECTRUM OF FIVE MINUTES.

Temperature		λ_m	$\lambda_m \times T$	J_m	$\frac{J_m}{T^5} \times 10^{14}$
304.0° C. =	577.0° Abs.	4.818	2780	1.546	2.421
190.5	463.5	5.964	2764	0.4083	2.105
99.9	372.9	7.393	2757	0.136	2.08

Radiating cavity of copper oxide.

The theoretical curve of formula (4) with $a = 5$ gives the above results from the observations. The product of $\lambda_m \times T$ is

[1] *Wied. Ann.*, 53, 301, 812, 1894.

[2] When this was let down an almost entirely closed cavity, containing a thermometer, presented itself to the slit. The intensity of its spectrum was computed for all the temperatures read and for all the wave-lengths, according to the principles of this research, and was then added to the observed intensity. The procedure is similar in testing the law of total radiation.

accordingly constant within the error of determination of λ_m,[1] but the intensity J_m of the wave-length λ_m of the maximum of energy, measured in arbitrary units, is not proportional to T^5 within the limits of error. Hence no certain conclusion can be drawn from these and similar measures.

BOLOMETER STRIP I, COVERED WITH A THICK LAYER OF LAMPBLACK, 5.0′ BROAD, AT THE CENTER OF THE POORLY REFLECTING HEMISPHERE.

Temperature		λ_m	$\lambda_m \times T$	J_m	$\frac{J_m}{T^5} \times 10^{14}$	
450.0° C. =	723.0° Abs.	4.009	2898	4.798	2.430	Lampblack in cavity[2]
. . .	. . .	. . .	. . .	. . .	. . .	
304.1	577.1	5.010	2891	1.547	2.416	Copper oxide in cavity
304.1	577.1	5.010	2891	1.544	2.402	
304.1	577.1	5.018	2896	1.567	2.438	
304.1	577.1	5.013	2893	1.553	2.419	Mean
. . .	. . .	. . .	. . .	. . .	. . .	
191.0	464.0	6.224	2888	0.5206	2.421	Copper oxide in cavity
191.0	464.0	6.216	2884	0.5260	2.446	
191.5	464.5	6.197	2879	0.5296	2.449	
191.2	464.2	6.212	2884	0.5254	2.439	Mean
. . .	. . .	. . .	. . .	. . .	. . .	
99.7	372.7	7.727	2880	0.176	2.45	Copper oxide in cavity
99.8	372.8	7.732	2883	0.173	2.40	
99.7	372.7	7.736	2883	0.175	2.43	
99.7	372.7	7.732	2882	0.175	2.43	Mean

The results of these measures show a distinctly better accord with the theoretical laws. Since the value of the product $\lambda_m \times T$ has been very greatly increased by the blackening of the bolometer, one cannot tell whether the arrangement already so far satisfies the theoretical postulates that this new value can be considered as valid within the errors of observation. This

[1] These errors are larger than in the later measures, as the course of the curve was not as accurately reproduced by formula (4) as with the black bolometers.

[2] In two further series, at 450° C., the layer of copper oxide in the cavity was too thin, so that it was somewhat transparent at short wave-lengths; and it was only with long waves, which glass emits strongly, that it was sufficiently black, so that I obtained larger values for $\lambda_m \times T$ (up to 2928).

value seems, moreover, to be slightly variable with the temperature. Further objections are discussed later. Therefore further measures were made in which the blackening of both bolometer and cavity was more perfect.

BOLOMETER STRIP II, 5.0′ WIDE, COVERED THICKLY WITH PLATINUM BLACK, IN THE POORER REFLECTING HEMISPHERE.

Temperature		λ_m	$\lambda_m \times T$	J_m	$\frac{J_m}{T^5} \times 10^{14}$	
450.0° C. =	723.0° Abs.	4.021	2907	3.707	1.877	Thick layer of copper oxide in cavity
450.0	723.0	4.018	2905	——	——	Same, but smaller aperture, covered with lampblack
450.6	723.6	4.007	2899	3.741	1.886	
	Mean	-	2904			
304.0	577.0	5.012	2892	1.200	1.869	Lampblack in cavity
191.0	464.0	6.223	2887	0.403	1.873	
[191.0	464.0	6.310	(2927)	0.393	1.826]	Glass cavity, with walls etched rough and not blackened [1]
Mean of all		-	2894 [2]			

This bolometer could not bear the strength of current necessary for investigating the temperature of 100° C.

BOLOMETER STRIP III, 6.3′ WIDE, BLACKENED WITH PLATINUM, IN THE PERFECT HEMISPHERE.

Temperature		λ_m	$\lambda_m \times T$	J_m	$\frac{J_m}{T^5} \times 10^{14}$	
449.5° C. =	722.5° Abs.	4.019	2903	7.492	3.807	Copper oxide in cavity
304.1	577.1	5.009	2891	2.446	3.805	
190.7	463.7	6.227	2887	0.806	3.76	Platinum black in cavity
190.2	463.2	6.240	2891	0.806	3.78	
100.5	373.5	7.722	2884	0.279	3.83	
	Mean	-	2892			

[1] At large wave-lengths glass emits as strongly as lampblack. Since it is only in a slight degree transparent at 4.5μ, a glass cavity cannot be used even at 100° C. One gets too great a wave-length for the maximum of energy, and at short wave-lengths the energy curve falls off too abruptly, in so far as the bands of the spectrum of the heating vapors do not shimmer through.

[2] The mean at 450° C. was calculated once, the value 2927 not at all.

BOLOMETER STRIP IV, 5.0′ WIDE, BLACKENED WITH PLATINUM, IN THE PERFECT HEMISPHERE.

Temperature		λ_m	$\lambda_m \times T$	J_m	$\frac{J_m}{T^5} \times 10^{14}$	
450.5° C.	= 723.5° Abs. (T)	3.998	2891	3.980	2.006	Lampblack in cavity
304.1	577.1	5.006	2889	1.219	2.012	
190.0	463.0	6.230	2885	0.422	1.984	Platinum black in cavity
190.7	463.7	6.230	2890	0.434	2.023	
99.8	372.8	7.742	2886	0.143	1.98	
	Mean - - -		2888			

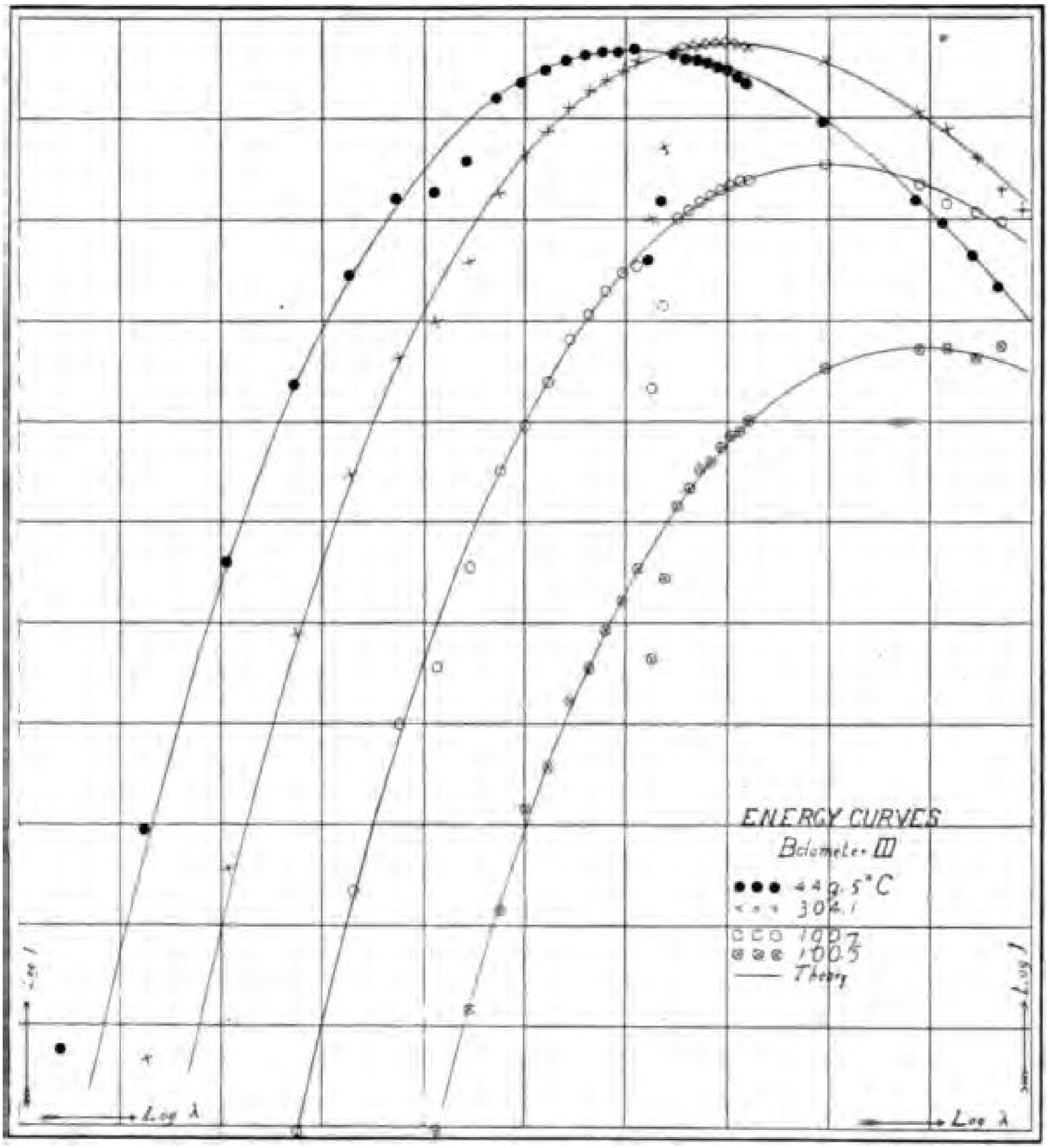
BOLOMETER STRIP V, 5.0′ WIDE, FIRST COVERED THICK WITH LAMPBLACK AND THEN WITH AN ELECTROLYTIC DEPOSIT OF PLATINUM BLACK, IN THE PERFECT HEMISPHERE.

Temperature		λ_m	$\lambda_m \times T$	J_m	$\frac{J_m}{T^5} \times 10^{14}$	
450.0° C.	= 723.0° Abs. (T)	3.997	2890	4.207	2.121	Lampblack in cavity
304.1	577.1	5.012	2893	1.361	2.117	
189.4	462.4	6.249	2889	0.449	2.123	Platinum black in cavity
100.0	373.0	7.745	2889	0.153	2.12	
	Mean - - -		2890	- -	2.120	

For judging of the accuracy of these results of the computations I would refer to the methods of reduction for the evaluation of the observed energy curves which are fully treated in my two previous papers. Here as there the most liberal use was made of the principle of the congruence of the energy curves represented logarithmically, which has been proven both theoretically and experimentally (Wien's third relation). Since formula (4) with $a = 5.00$ always reproduced the curve within the limits of error, excepting two or three unreliable extreme points at small wave-lengths, the value of λ_m and J_m could be determined with very great accuracy with a uniform regard of all the observed points. The figure gives the logarithmic representation, with $\log J +$ const. as ordinate and $\log \lambda$ as abscissa, of four energy curves obtained with bolometer III. The full line represents the curve of the formula (4) with $a = 5.000$, as it most closely fits the observations. The congruence is seen at once from the fact that this line is the same for all four curves.[1]

[1] In *loc. cit.* the energy curves were drawn one above the other in order to prove this.

Only the extreme points of short wave-lengths, where the measured energy at most amounts to $\frac{1}{100}$ of the maximum energy, deviate in the direction to cause a suspicion of diffuse light. This is entirely possible, since the maximum of energy of the observed prism-spectrum lies very near these points.



In order to exhibit the validity of law I in still another way, I have put together as "isochromatic lines" the intensities observed for the same wave-lengths from the four energy curves of the last series of measures (bolometer V), by treating $\log J$ as a function of $\frac{1}{T}$ for each wave-length. This was possible for the reason that observed points of these series lay nearly at the same wave-lengths. An interpolation in the course of the curve would have been less convincing. Within the errors of measurement the isochromatic lines thus obtained were straight lines, and the values of c_1 and c_2 of formula I calculated from them agree, as was to be expected, with the more precise values of these constants given by the evaluation of the observations as energy curves, or hence given by the above tables.

The latter values are:

$$c_2 = \lambda_m \times T \times 5 = 2890 \times 5 = 14450$$

$$c_1 = \frac{J^m}{T^5}(\lambda^m \times T)^5 \times e^5 = 2.120 \times 10^{-14} \times 2890^5 \times 2.7183^5 = 634100.$$

The following table contains the summary of the observed values of log J included for the isochromatic lines. The calculation of the constants of the straight line for each wave-length yields the values of c_1 and c_2 given below them, the means of which (omitting the line at 1.887μ) are

$$c_2 = 14450, \qquad c_1 = 629100.$$

Since only a small number of observed points were included here, while all the observed points were taken into account in the calculation of the energy curves, we could not expect a better agreement in the value of c_1. The values derived from the energy curves are of course more accurate.

SUMMARY OF CERTAIN OBSERVED POINTS LOG J FOR ISOCHROMATIC LINES (BOLOMETER V).

T (Abs.)	$\frac{1}{T}$	λ=7.783	6.263	4.663	3.355	2.280	1.887μ
	0.00						
373.0	2681	0.184–1	0.132–1	0.846–2	0.155–2	—	—
462.4	2162	0.599–1	0.647–1	0.548–1	0.1305–1	0.043–2	
577.1	1733	0.951–1	0.0826	0.1240	0.9311–1	0.236–1	0.669–2
723.0	1383	0.2407	0.4375	0.5954	0.5775	0.1869	0.825–1
	c_2=	14440	14460	14450	14450	14450	14360[1]
	c_1=	629400	636900	630300	629600	613700	738800[1]

[1] These points at the extreme end of the rising branch are also here exhibited as lying too high (c_1 too large), and due to light of longer wave-length (c_2 too small.)

There is still one last consequence of the observations to be adduced in forming a judgment of the results obtained with the different arrangements of bolometers. I could not know on general principles whether a disturbing absorption of the air of the room would be present in addition to the absorption of the prism, in the region from 7.7μ to 10μ. As this subject has not been sufficiently investigated, since it was desired to include this region for the temperature 100° C., I have determined the factors for curves of higher temperature which were safely located by observations at short wave-lengths, with which factors the observed intensities beyond 7.7μ are to be multiplied in order that they should fit the theoretical curves. The factors thus determined therefore correct everything that would tend to make the observed intensities too small. I have on the other hand calculated by the law of absorption the factor which alone eliminates the absorption of the prism. It appeared that the factors for the blackest bolometers determined solely from the depression of the energy curve agreed thoroughly with the factors which corrected the absorption of the prism, so that it is improbable that there is any further absorption of any extent at the wave-lengths here observed. Lampblacked bolometer I in the poor hemisphere demands on the other hand still larger factors, which deviate increasingly with increasing wave-lengths, thus rendering necessary the conclusion that in this region this bolometer strip has less absorptive power than the others. The two following tables give information as to this:

I. DETERMINATION OF THE FACTORS FROM THE DEPRESSION OF THE ENERGY CURVE.

$\lambda =$	7.738	8.245	8.807	9.324
Bolometer I[1]	1.275	1.645	2.695	4.968
II	1.213	1.536	2.420	4.288
III	1.202	1.567	2.353	4.181
IV	1.205	1.563	2.388	4.436
V	1.217	1.555	2.449	4.454
Mean of II to V	1.209	1.555	2.403	4.340

[1] The energy curves of this bolometer are corrected with these factors.

2. CALCULATION OF THE FACTORS FROM THE ABSORPTION OF THE PRISM.

If the layer of fluor-spar of thickness l absorbs the fraction a, the emergent radiation from a prism of base B must be multiplied by the factor

$$\frac{\log (1-a)^{\frac{B}{l}}}{\left\{(1-a)^{\frac{B}{l}}-1\right\} \log e}$$

to obtain the incident radiation, as may be seen from a simple integration based on the law of absorption. The prism had its corners chipped, and had by no means a mathematically simple shape[1]. Measurement of the base gave in the mean a value of 43.0 mm. In the following table δ is the minimum deviation of the prism, λ the corresponding wave-length, and a the amount of the absorption I observed in a clear plate of fluorite of 4.056 mm thickness. The factor computed is calculated from the absorption, the factor observed is that determined from the energy curves above.

δ	λ	a	Factor comp.	Factor obs.
25° 19.1′	7.738μ	0.0365	1.211	1.209
24 29.1	8.245	0.0885	1.570	1.555
23 29.1	8.807	0.1805	2.404	2.403
22 29.1	9.324	0.3125	4.042	4.340

Not until $\lambda = 9.324\mu$ does the observed intensity have to be increased more than corresponds to the absorption of the prisms. This may be due to the absorption of the air of the room or to an erroneous determination of the absorption of the fluor-spar, but it has no bearing on our conclusions. The energy curves obtained with an ordinary lampblacked bolometer must lie too low even at short wave-lengths, representing a power of absorption of the lampblacked surface decreasing for increasing wave-lengths. The layer of lampblack (through which the radiation has to pass twice) is therefore still considerably transparent even

[1] In *Wied. Ann.*, 53, 333, 1894, I give the observations of the total loss of light, from which 5.05 per cent. is to be deducted for loss by reflection.

in a thick stratum, or else it has a rising reflecting power with increasing wave-length.[1] From several experiments the latter seems to me to be the case. Any such deviation at these long wave-lengths would be naturally expected, since the absorption of the lampblack bolometer strip in the reflecting shell can only be approximately equal to unity. Although the arrangement of bolometers II to V was different and represented an increase of the blackening (passing from the imperfect to the perfect hemisphere), nevertheless they absorbed almost equally in this spectral region, so that they seem to justify the conclusion that they all approximate "the absolutely black body" in this region so nearly that the casual deviation cannot produce appreciable error. I therefore believe that it is permissible to employ the results of the measurements with these bolometers in the determination of the constants of law I.

The constant c_2 is most accurately determined from these experiments by the fivefold value of the constant product $\lambda_m \times T$. This is in the mean:

For bolometer	II	III	IV	V	Mean of all.
	2894	2892	2888	2890	2891

The single values of a series of observations which deviate most widely from the mean of all are:

2907 (Bolometer II, 450°, cavity with layer of copper oxide);
2884 (Bolometer III, 100.5°, cavity with layer of platinum-black).

The first value is not entirely reliable, since it easily happened that some parts of the layer of copper oxide fell off, leaving the glass uncovered. This always gives occasion for an increase of the wave-length of the maximum.

I estimate the highest possible error of the mean of all as something like 3 per thousand.

Accordingly c_2 will be $= 5 \times 2891 = 14455$, with an error which at most I estimate at 40.

Since in formula I, $\frac{c_2}{\lambda T}$ represents a number, c_2 must have the

[1] K. Ångström has observed in thin layers an increasing transparency of lampblack with increasing wave-length. *Wied. Ann.*, **36**, 720, 1889.

dimension: Wave-length × Temperature. Therefore

$$c_2 = 14455 \times \text{Degree Centigrade of absolute scale.}$$

The value of c_2 found by H. Wanner and myself[1] in a different region of temperature and wave-length by measurements of an entirely different kind was 14440, but it had considerably less accuracy.

The bolometric measurements at high temperatures recently published by O. Lummer and E. Pringsheim[2] yielded values of $\lambda_m \times T$ from 2837 to 2928, mean 2879, for one series, and for another series, in which the exponent 5.2 held good in the general formula I, five values between 2766 and 2986, mean 2876. From these they deduce for values of c_2, 14395 from the first series, and 14955 from the second series. Taking into account the errors possible according to the results of the observations and to the methods of reducing them, the agreement with my value seems to be good.

F. Kurlbaum[3] has measured the difference of the total radiation of a cavity coated with lampblack at 100° C. and at 0° in Watts × cm^{-2}, and has calculated that the heat equivalent of the total radiation emitted by a black body at the absolute temperature T amounts to

$$1.277 \times 10^{-12}\, T^4 \frac{\text{Gr. Cal.}}{\text{cm}^2 \times \text{sec.}}$$

or

$$5.32 \times 10^{-12} \times T^4 \text{ Watts} \times \text{cm}^{-2}.$$

By integrating formula I we get[4]

$$\int_0^\infty J\, d\lambda = 6 \frac{c_1}{c_2^4} T^4 \qquad \text{(II)}$$

The constant c_1 has by formula I or formula II the dimensions

$$\text{Energy of radiation} \times (\text{Wave-length})^4.$$

Their numerical result depends, therefore, on the unit of the

[1] This JOURNAL, 9, 304, 1899.

[2] *Verhandlungen der Deutschen Physikalischen Gesellschaft*, February 3, 1899.

[3] *Wied. Ann.*, 65, 746, 1898.

[4] *Loc. cit.*, p. 666, where for $\alpha = 5$, $\Pi(\alpha - 2) = 6$.

energy of radiation. In my experiments this was arbitrary and was different for the six different bolometers, so that a more precisely definite value of the quantity c_1 could not be computed from these experiments alone. In order to refer the measurements to a definite unit of radiation let us now determine the value of c_1, with which the total radiation of my source of radiation reaches the quantity determined by Kurlbaum, so that the energy of radiation is measured in

$$\frac{\text{Gr. Cal.}}{\text{cm}^2 \times \text{sec.}} \text{ or } \frac{\text{Watts}}{\text{cm}^2}. \quad \text{From the relation}$$

$$\underbrace{6\frac{c_1}{c_2^4}T^4}_{\text{Second member of II}} = \underbrace{1.277 \times 10^{-12}\, T^4 \frac{\text{Gr. Cal.}}{\text{cm}^2 \times \text{sec.}}}_{\text{Kurlbaum's Heat-equivalent}}$$

it follows that with the value $c_2 = 14455 \times$ Temperature degrees that

$$C_1 = 9292 \frac{\text{Gr. Cal.}}{\text{cm}^2 \times \text{sec.}} \times \mu^4,$$

or

$$38710 \frac{\text{Watts}}{\text{cm}^2} \times \mu^4.$$

With these values of the constants the equivalent in work for the radiation of the black body of Temperature T, for any region of wave-length whatever, may therefore be computed by formula I.