ON THE DISTRIBUTION OF ENERGY IN THE SPECTRUM OF THE BLACK BODY AT HIGH TEMPERATURES.[1]

By F. PASCHEN.

I WAS able to show in a previous communication[2] that the law of radiation of the black body proposed by W. Wien is the more completely confirmed by observations, within the range of temperature from 100° to 450° C., according as the arrangement of the experiment is more closely adapted to the theoretical premises. This law has also received an important support from the theoretical researches of M. Planck.[3] He derives the law from his electro-magnetic theory of radiation, and thereby introduces only one additional hypothesis, that for the definition of Entropy, which, although not regarded by him as fully established, is, however, probably more reliable than the assumptions which led Wien to the same results based on the kinetic theory of gases. According to Planck's presentation also, the law appears to be a rigorously valid law of nature, and its contents possess a general significance.

The problem remaining for experimental investigation is, first, to investigate the limits within which the law, and hence Planck's assumptions, are found to hold good; and second, to determine the constants of the law as accurately as possible.

From the results of the investigation of the region of lower temperatures I expected to find the law to be valid within very small limits of error at higher temperatures. My experiments showed, however, that many kinds of experimental difficulties arise here.

[1] *Sitzungsberichte der Berliner Akademie.* Session of the Physical-Mathematical Section on December 7, 1899.

[2] This JOURNAL, 10, 40, 1899.

[3] "Ueber irreversible Strahlungsvorgänge." *Sitzungsberichte der Berliner Akademie*, Session of May 18, 1899, p. 440.

While at low temperatures the production of radiation by the black body appears quite simple, and only a few special adaptations of the bolometer are necessary in order that the radiation of the longer wave-length shall be sufficiently absorbed, it was comparatively difficult for me to realize the radiation of the black body so perfectly at higher temperatures that its energy curve followed the law within the limits of the errors of observation.

I attempted to produce the radiation first by cavities with heated walls, and secondly, by causing the total radiation from a small incandescent surface to be repeatedly reflected by mirrors upon the surface itself. In the first arrangement, which seems to be the better, it is extremely difficult to heat with sufficient uniformity the walls of the cavity provided with an aperture. The greater the cavity and its aperture, and the higher the temperature, so much greater will be the difference in temperature at different points of the walls. Not until these differences were reduced to a small amount did the emergent radiation exhibit the energy curve of Wien's law with a sufficient degree of approximation.

With the second arrangement,—that of producing the radiation of the black body by reflection,—it is, indeed, possible to heat a small opaque surface uniformly, but the difficulty is to make the reflection somewhere nearly complete. Even if we can only correct by this the deviation[1] of the radiation of the surface from that of the black body at the same temperature,—a difference which can be reduced to a small amount by the use of a suitable surface for the incandescent layer,—I should nevertheless have been obliged to use an expensive reflecting hemisphere. The form of the image outside of the center of a reflecting sphere introduces further an irregularity, varying with the temperature, in the incandescence of a surface which is not very small. Finally the accurate measurement of a high temperature turns out be very difficult with this arrangement.

On gradually improving the arrangement of the experiment, according to both methods, I have found that each improvement

[1] *Wied. Ann.*, 60, 719, 1897.

reduces the amount of the departure of the results of the observations from the demands of theory, until finally results were obtained by both methods in complete agreement among themselves and also apparently in sufficient agreement with the results obtained at lower temperatures. The slight isolated deviations from the law found in the results now to be communicated are for the most part the relics of larger departures, and therefore only prove that certain defects in the arrangement were not sufficiently overcome. Should we desire to carry the accuracy of the measurements still further, however, it would be necessary to make a decided improvement in the arrangement, now only just sufficient, in order to establish the validity of the law within still smaller limits of error.

As it would lead us too far aside, if I were to detail all the researches which I have made in order to surmount the difficulties of a purely practical nature, I shall limit myself to the communication of the results obtained with the most suitable arrangements employed for the purpose.

The general arrangement of the spectroscopic apparatus remained the same as for the investigation at lower temperatures. In all the measures at high temperatures the exposed bolometer filament consisted of a strip of platinum 4 mm long, 0.3 mm wide, and $\frac{1}{2000}$ mm thick, which was covered with a thick layer of platinum black, and was placed at the center of the reflecting hemisphere previously described. The current in the bolometer was always maintained at the same strength, and the sensitiveness of the apparatus to radiation was varied by the introduction of resistances in the circuit of the galvonometer.

The temperature was always measured with a thermo-element of wires of platinum and platinum-rhodium of 0.15 mm diameter, which Mr. Holborn had kindly calibrated for me in 1898. All temperatures above 450° C. correspond to this calibration, but lower temperatures correspond to the scale of two mercury thermometers which had been certified by the *Reichsanstalt*. The thermo-electromotive forces were measured by compensation.

In measuring up an energy spectrum at those places where the energy rapidly changed (at small wave-lengths) I proceeded by steps of the apparent breadth of the bolometer strip (3′), in order to be able to accurately apply the correction for the impurity of the spectrum caused by the width of the bolometer strip and the equally wide slit. In the remaining part of the spectrum I selected a few places as free as possible from absorption and uniformly distributed. Along with the energy curve thus more completely observed I made check measurements at the same temperatures, determining the throw of the galvanometer at only eight or ten places, uniformly distributed in the spectrum. These served as a check on the more complete energy curves and for the most part they are not given in the tables.

The observations were at first compared with the theory in the same way as in my paper on lower temperatures. For this purpose the observed values of the wave-length λ_m and of the intensity J_m of the maximum of the energy curve were given in tables, and the relations obtained for these constants were tested, *viz.*:

I. $\lambda_m \, . \, T = \text{constant.}$

II. $J_m / T^5 = \text{constant.}$[1]

In case the observed energy-curve differs appreciably from the theoretical requirement, *viz* :

III. $$J/J_m = \left\{ \frac{\lambda_m}{\lambda} e^{\frac{\lambda - \lambda_m}{\lambda}} \right\}^5 ,$$

this is indicated in a remark. In order to show how far the observed energy-curve follows this law, the observed points of

[1] In the determination of the values λ_m and J_m from an energy curve, all of the observed points of the curve are always employed, so that, $\lambda_m \, . \, T = \text{constant}$ at the same time expresses the fact that each wave-length is so displaced that the relationship $\lambda \, . \, T = \text{constant}$ is also fulfilled. In case relation II holds, the intensities of the wave-lengths therefore corresponding with each other are further proportional to T^5 (Wien's general law). The energy curves at different temperatures, expressed in logarithmic measure (with $\log J$ for ordinates and $\log \lambda$ for abscissas) are accordingly congruent, or J/J_m is the same function of λ/λ_m at all temperatures. This relation, together with I and II above, is identical with the relations as formulated by Wien.

different energy-curves are represented in a figure, along with the theoretical curve III. If the three conditions I, II, and III are fulfilled, Wien's general formula

$$\text{IV.} \qquad J = c_1 \lambda^{-5} e^{-\frac{c_2}{\lambda T}}$$

is established. A more rigorous mode of investigating the law will be given in my conclusion.

I. MEASUREMENTS ON CAVITIES WITH HEATED WALLS.

A cavity of unglazed porcelain was made in the manner described by Lummer and Kurlbaum.[1] A porcelain cylinder of 5 cm diameter and 10 cm length having six cross walls was made in one piece. The length of the central radiating cavity was 5 cm, and it had two chambers both before and behind it. The apertures of the three walls in front had an area 0.6 sq. cm. A cylinder of platinum foil was wrapped around the porcelain cylinder, and was heated by an electric current. The radiating body was surrounded by an air space and by an asbestos covering. The losses of heat by conduction, chiefly due to the apertures, cause a lack of uniformity in the temperature of the walls of the cavity. The wall of the cylinder is hotter than the rear wall, and this again is hotter than the front wall, which is especially cooled off in the immediate neighborhood of the aperture. Without a protecting cover, the upper half of the radiating body is hotter than the lower half, in consequence of the rising warm air, and those parts of the platinum foil which are in contact with the porcelain differ from those which are not in contact. These irregularities increase with rising temperature, and further depend upon the external protection of the radiating body and upon the diaphragms placed in front of the aperture.

The thermo-element was introduced through the rear cross walls, and measured the temperature of the radiating part of the rear wall. As soon as the incandescent body has reached a stationary condition, the thermo-element almost completely vanishes in the background, even if very considerable difference in

[1] *Ber. der Phys. Gesellschaft zu Berlin*, 17, 106, 1898.

the temperature occur in the walls,—which may probably be explained by the near equality in the reflecting power for visible rays of the metal wires and of the porcelain.

In the three following series of measures the cavity was differently protected, and the light coming from the aperture was differently diaphragmed. For a given series the arrangement was kept as nearly as possible the same.

LARGE PORCELAIN CAVITY.

TABLE I

Temperature C.	Temperature Abs.	$\lambda_m (\mu)$	$\lambda_m T$	J_m	$J_m/T^5 \times 10^{15}$	
413°7	686°5	4.222	2899	0.5420	3.553	
443.2	716.2	4.057	2905	0.6740	3.578	
643.8	916.8	3.199	2933	2.320	3.582	
693.0	966.0	3.044	2941	3.041	5.612	
907.3	1180.3	2.492	2941	8.464	3.695	Ch.[1]
933.0	1206.0	2.437	2940	8.560	3.764	
1048.1	1321.1	2.233	2950	15.03	3.737	Ch.
1053.4	1326.4	2.219	2945	15.65	3.807	
			Mean 2932	m. e. = 5		

TABLE II

411°6	684°6	4.235	2899	0.5114	3.398	
692.8	965.8	3.033	2928	2.855	3.397	
865.5	1138.5	2.556	2911	6.483	3.383	At the longest wave-lengths higher than the theoretical curve.
871.2	1144.2	2.575	2946	6.635	3.389	
958.2	1231.2	2.381	2931	9.528	3.370	Ch. Fits the theoretical curve poorly.
			Mean 2923	m. e. = 8		

TABLE III

374°1	647°2	4.508	2917	0.5419	4.774	Ch.
377.3	650.3	4.446	2889	0.5543	4.767	
654.0	927.0	3.155	2925	3.265	4.768	Ch.
664.6	937.6	3.113	2919	3.513	4.850	
845.2	1118.2	2.610	2918	8.534	4.888	
1043.4	1316.4	2.218	2921	18.79	4.741	Inaccurate on account of unsteady temperature.
			Mean 2915	m. e. = 5		

[1] Ch. denotes a check measure as distinguished from the more complete measures.

The third of this series may probably be regarded as the best in respect to arrangement. The differences in the values of $\lambda_m \cdot T$ and J_m / T^5 are larger in series I than would be expected from the residuals.

In order to obtain a more uniform temperature of the cavity, I selected the arrangement sketched in Fig. 1 *a* and *b*. A metallic crucible *T* (of copper or platinum), with thick walls,

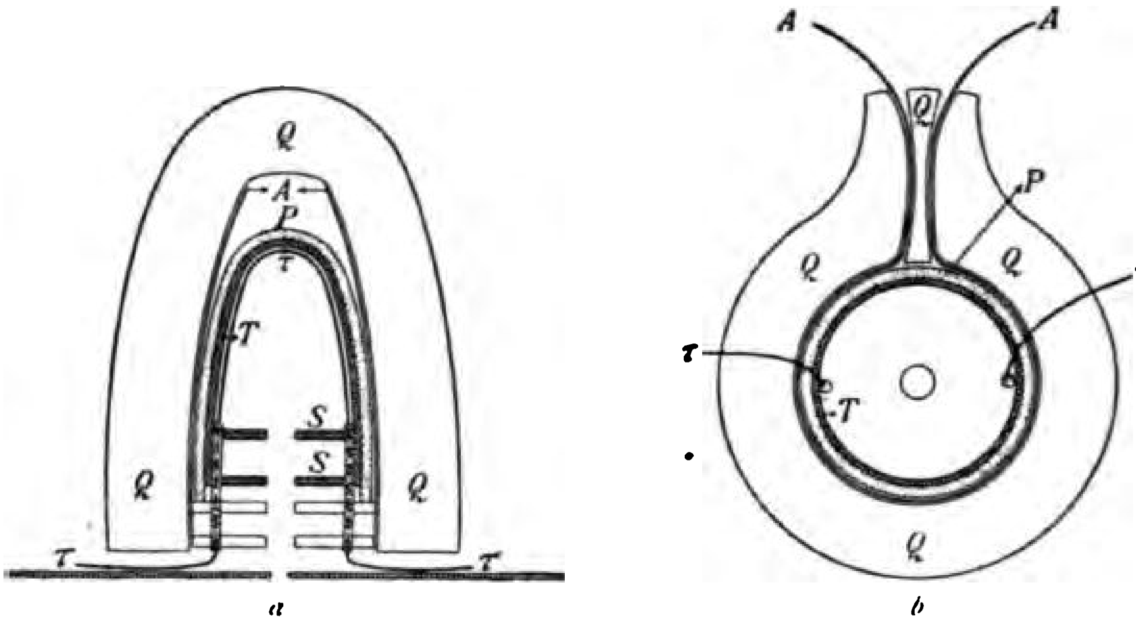

FIG. 1.

was separated into two chambers by two round separating walls *S*, of the same metal, with small round apertures. The rear chamber was the radiating cavity. A porcelain crucible *P*, with very thin walls, surrounded the metallic crucible. For the electric heating of the space I wrapped about this, as may be seen from the drawing, the platinum foil *A* which projected out from in front and from behind. *Q* was an asbestos cover. Two or three cross walls of asbestos were also attached. The length of the cavity amounted to about 3.5 cm, with an extreme breadth of 3 cm. The round apertures in *S* had a diameter of about 5 mm. As a result of the good conductivity in the metallic wall, and of the small dimensions of the cavity, a pretty uniform temperature was obtained in the interior of the cavity. The thermo-element *T* was introduced from in front through a thin small tube of porcelain. When the inner surface of the wall was composed of metallic oxides (of copper or iron) the thermo-element did not disappear in the background unless the temperature of the cavity was practically uniform throughout, since the reflection of the visible rays was very different from the platinum wires and from the metallic oxides. It appeared dark on a bright ground when the metallic walls *S* were cooler than the back part of the crucible, and bright on a dark ground when they were warmer. The temperature distribution in the cavity could, however, be altered according to the mode of the external wrapping with asbestos, and it was possible to regulate this in such a way that the thermo-element almost entirely disappeared.[1]

If the interior metal wall was surrounded with porcelain, however, this appearance ceased to give a sensitive mode of judging of the inequalities of temperature in the cavity.

First arrangement. The metal crucible was of copper, which oxidized on incandescence in the air, and was almost entirely converted into oxide at the close of the experiment.

TABLE IV

Temperature C.	Temperature Abs.	λ_m	$\lambda_m \cdot T$	J_m	$J_m : T^5 \times 10^{15}$	
560°0	833°0	3.482	2900	1.583	3.946	
885.3	1158.3	2.525	2925	8.410	4.035	
501°2	774°2	3.748	2901	0.8532	3.069	Apertures smaller.
685.2	958.2	3.067	2939	2.464	3.050	
878.7	1171.7	2.502	2931	6.669	3.021	
			Mean 2919	m. e.= 8		

Second arrangement. The metal crucible was of platinum. The inner surface of the cavity was covered with a thin wall of porcelain. (A second porcelain crucible was fitted into the platinum crucible. The front cross wall of the cavity consisted of porcelain on the inside and platinum on the outside.)

[1] The wires could no longer be seen by an observer who did not know their location.

TABLE V

Temperature C.	Temperature Abs.	λ_m	$\lambda_m \cdot T$	J_m	$J_m/T^5 \times 10^{15}$	
411°.1	684°.1	4.212	2882	0.2015	1.344	Ch.
731.6	1004.6	2.896	2910	1.380	1.349	
732.0	1005.0	2.891	2904	1.394	1.360	Ch.
733.7	1006.7	2.884	2903	1.401	1.355	Ch.
1029.1	1302.1	2.233	2908	5.286	1.413	Ch.
1035.7	1308.7	2.223	2910	5.237	1.363	
1044.3	1317.3	2.217	2920	5.534	1.396	Ch.
1283.9	1556.9	1.887	2939	12.80	1.399	
			Mean 2910	m. e. = 6		

Third arrangement. The platinum crucible of the previous arrangement was covered with iron oxide after the removal of the porcelain wall.

TABLE VI a

Temperature C.	Temperature Abs.	λ_m	$\lambda_m \cdot T$	J_m	$J_m/T^5 \times 10^{15}$	
743°.0	1016°.0	2.858	2899	1.514	1.398	
1060.5	1333.5	2.188	2917	5.984	1.419	
467.6	740.6	3.864	2862	0.3221	1.446	
500.6	773.6	3.713	2873	0.4042	1.458	Ch.
687.4	960.4	3.010	2891	1.208	1.478	
731.4	1004.4	2.865	2878	1.507	1.474	Ch.
951.4	1224.4	2.367	2898	4.134	1.503	
961.2	1234.2	2.357	2908	4.331	1.508	Ch.
979.5	1252.5	2.310	2893	4.578	1.484	
982.0	1255.0	2.312	2902	4.639	1.489	Ch.
1294.3	1567.3	1.849	2898	14.39	1.523	At long wave-lengths deviations from the theoretical curve occur.
423.2	696.2	4.153	2891	0.4088	2.501	
737.5	1010.5	2.861	2891	2.795	2.655	
762.5	1035.5	2.791	2890	3.235	2.719	
791.6	1064.6	2.721	2897	3.632	2.655	The observed points lie above the theoretical curve at both ends of the curve.
1008.4	1281.4	2.271	2909	9.438	2.734	
1221.	1494.	1.967	2941	20.08	2.690	The curve is too high at the longer wave-lengths.
			Mean 2896	m. e. = 4		

The platinum crucible was freshly covered with iron oxide; aperture very small and incandescence very uniform.

TABLE VI b

Temperature C.	Temperature Abs.	λ_m	$\lambda_m \cdot T$	J_m	$J_m/T^5 \times 10^{15}$	
701°.4	974°.4	2.960	2886	1.726	1.963	Ch.
706.3	979.3	2.951	2890	1.767	1.961	
992.7	1265.7	2.286	2893	6.557	2.011	
995.6	1268.6	2.282	2895	6.647	2.023	Ch.
1019.7	1292.7	2.246	2905	6.920	1.915	Ch.
1259.4	1532.4	1.907	2921	16.62	1.966	Ch.
1260.3	1533.3	1.906	2921	16.79	1.981	
			Mean 2901	m. e. = 5		

II. MEASUREMENTS OF THE RADIATION OF THE BLACK BODY BY THE METHOD OF REFLECTION.

The following arrangement of the incandescent body was chosen for the experiments now to be described (Fig. 2, *a*, *b*, *c*). The radiating sheet of platinum *P*, of 0.05 mm thickness, and of about 30 mm length and 16 mm width, and two shorter sheets of platinum *S*, insulated by mica *G*, are wrapped about a strip of platinum foil or platinum-iridium foil *A*, which heats *P* and *S*, being itself heated by an electric current. The thermo-element *T* is insulated by mica from the sheet *S* and from the parts of *P* furthest from the middle, and touches *P* with the junction from the inside, that is, is attached with the junction at the middle of *P*.[1]

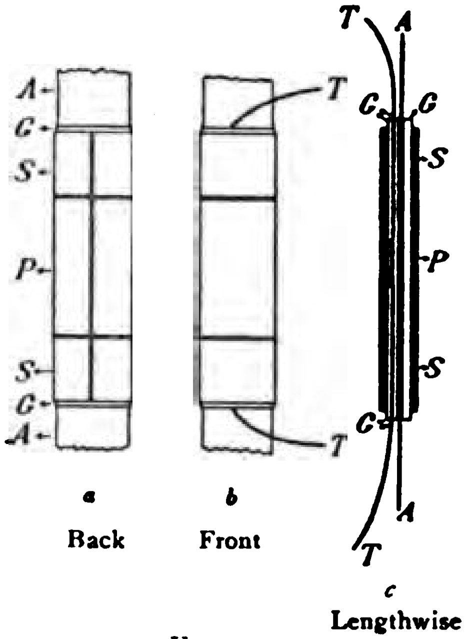

FIG. 2.

The piece *P* becomes uniformly incandescent as soon as a high temperature has been once suddenly produced, at which the water present in the mica quickly vaporizes and thereby

[1] In some of the experiments the junction was situated in the exterior surface, so that the conducting wires, insulated by a coating of iron oxide, passed through *P*; or the ends of the thermo-element which were not soldered together were allowed to touch *P* from within. The indications of temperature up to 800° C. were almost the same in all these arrangements.

uniformly inflates the strip of platinum P. The front surface of P was so adjusted in the central plane of a reflecting hemisphere that it was covered as closely as possible by its reflected image. The reflecting surface had a diameter of 15 cm and consisted of German silver; it had a good spherical figure and was admirably polished. It had a small aperture toward the incandescent surface P through which the radiation fell on the slit. The dimensions of the incandescent surface P are larger than was necessary to fill the diaphragm in front of the prism of the spectroscopic apparatus, on account of the imperfections of the image. The radiation from a portion of P only about 10 mm × 20 mm reaches the bolometer. In so far as the radiation of this smaller surface is reflected on neighboring parts of P and not on the surface itself, in consequence of the imperfections of the image, the radiation of these neighboring portions does, nevertheless, fall back upon the central portion. With a uniform incandescence of P this arrangement is accordingly equivalent to that in which the small surface only is present, and receives its own radiation completely returned upon itself.

First arrangements (for lower temperatures). The radiating sheet of metal is covered with platinum black. A bolometer strip 6′ wide is employed in the reflecting hemisphere. (See my paper on lower temperatures).

TABLE VII

Temperature		λ_m	$\lambda_m \cdot T$	J_m	$J_m / T^5 \times 10^{14}$	
C.	Abs.					
99°.9	372°.9	7.730	2883	0.2737	3.797	
186.9	459.9	6.251	2875	0.7805	3.793	
190.7	463.7	6.211	2880	0.8136	3.797	
192.1	465.1	6.194	2883	0.8248	3.788	Larger deviations [1]
313.2	586.2	4.926	2887	2.620	3.787	
316.7	589.7	4.899	2890	2.683	3.761	
318.8	591.8	4.881	2889	2.730	3.760	
453.6	726.6	3.985	2896	7.642	3.773	
599.4	872.4	3.327	2902	18.93	3.747	Higher at the ends than the theoretical curve [1]
603.2	876.2	3.312	2901	19.56	3.787	
618.3	891.3	3.252	2899	21.30	3.760	Somewhat too high at the ends [1]
624.1	897.1	3.232	2900	21.93	3.773	
			Mean 2890	m. e. ‒ 3		

[1] At the higher temperatures the sheet is somewhat irregular in its incandescence.

Second arrangement. The bolometer strip of 6′ breadth is replaced by one of 3′. The radiating body is the same as in the first arrangement.

TABLE VIIIa

Temperature		λ_m	$\lambda_m \cdot T$	J_m	$J_m / T^5 \times 10^{15}$
C.	Abs.				
387°.0	660°.0	4.380	2891	0.5923	4.742
400.8	673.8	4.290	2891	0.6591	4.742
418.7	691.7	4.178	2890	0.7498	4.756
419.1	692.1	4.176	2891	0.7647	4.780
549.1	822.1	3.531	2904	1.783	4.743
557.3	830.3	3.495	2912	1.875	4.753
574.9	847.9	3.424	2903	2.095	4.779
633.5	906.5	3.214	2913	2.926	4.781
			Mean 2899	m. e. = 3.5	

Third arrangement. The platinum strip is covered with oxide of copper.

TABLE VIIIb

Temperature		λ_m	$\lambda_m \cdot T$	J_m	$J_m / T^5 \times 10^{15}$
C.	Abs.				
560°.5	833°.5	3.488	2907	1.907	4.737
643.1	916.1	3.174	2908	3.095	4.797
656.9	929.9	3.126	2907	3.331	4.792
737.8	1010.8	2.877	2908	5.089	4.827
848.	1121.	2.596	2908	8.494	4.799
861.	1134.	2.559	2901	8.874	4.732
			Mean 2906	m. e. = 1	

The following measurements were obtained with a somewhat different adjustment of the radiating body and of the bolometer, and with a sheet of platinum P which was not uniformly incandescent in the reflecting sphere:

Temperature		λ_m	$\lambda_m \cdot T$	J_m	$J_m / T^5 \times 10^{15}$	
C.	Abs.					
387°.2	660°.2	4.367	2883	0.6084	4.856	Observed curve is too high at the ends.
646.6	919.6	3.159	2905	3.196	4.859	
868.2	1141.2	2.565	2926	9.454	4.882	

A strip covered with iron oxide and uniformly incandescent gave:

Temperature		λ_m	$\lambda_m \cdot T$	J_m	$J_m/T^5 \times 10^{15}$	
C.	Abs.					
756°5	1029°5	2.817	2900	5.655	4.891	

It is very difficult to obtain a uniform incandescence of a larger surface at the higher temperatures with the arrangement described, if an image of the surface itself is reflected upon itself by the hemisphere. The central parts of the surface receive a greater amount of reflected radiation, and are better protected from loss of heat than the edges. I have obtained a fairly uniform condition of incandescence in the reflecting envelope with a surface 10 mm wide of platinum 0.1 mm thick, when the back side of the strip was protected from loss of heat by a cover of mica or asbestos. Outside of the center of the reflecting sphere the strip is then less incandescent than at the edge, and if its image is projected upon itself it will be uniformly incandescent at one definite temperature only. The strip to which the following results refer was most uniformly incandescent at about 1000° C. With this arrangement I could no longer obtain a reliable determination of the temperature with the thermo-element, since the mica insulation of the wires is insufficient at 1000°. For one set of energy curves I have determined the temperatures which seem to satisfy most closely laws I and II. The curves had very nearly the shape demanded by theory.

Fourth arrangement. A surface of 10 × 25 mm, blackened with iron oxide; the diaphragm in front of the prism smaller.

TABLE IX

Abs. Temp. assumed	λ_m (obs.)	$\lambda_m \cdot T$	J_m (obs.)	$J_m/T^5 \times 10^{15}$	
834°9	3°468	2895	1.134	2.796	
1014.	2.862	2900	3.015	2.821	
1021.5	2.835	2896	3.129	2.815	
1273.	2.280	2903	9.478	2.833	The last end of the curve at long wave-lengths is a little too high.
1496.	1.945	2910	21.47	2.866	

It should be further mentioned that measurements, showing tolerable agreement with the experiments described and with the law, were made with a reflecting hemisphere of bronze, and with a different arrangement of the incandescent strip of platinum. As the arrangement was, however, defective in many respects,

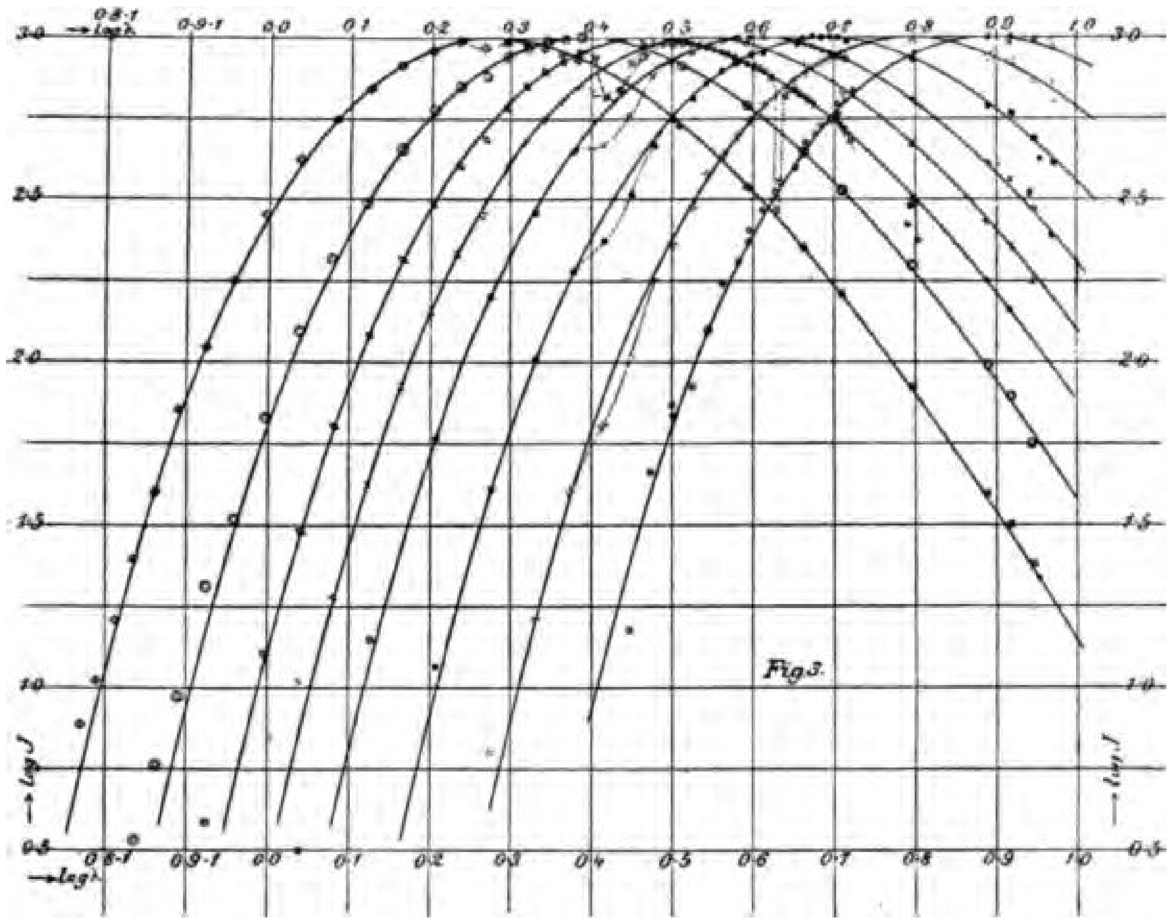

FIG. 3.

these measures need not be taken into consideration in comparison with those described.

It follows from the preceding results that the relations I and II have been found to hold good with a pretty close degree of approximation with most of the arrangements described. Somewhat larger deviations do indeed occur in a few of the series.

The observed points of a series of energy curves are represented with log J as ordinate and log λ as abscissa, in Fig. 3, and their symbols are given in the following list. For convenience of inspection log J_m is made the same in all the curves.

LIST OF THE ENERGY CURVES REPRESENTED IN FIG. 3.

Temp. C	λ_m	log λ_m	Table	Symbol
99°.9	7.730	0.8882	VII	⊚ ⊚ ⊚
190.7	6.211	0.7932	VII	× × ×
318.8	4.881	0.6885	VII	• • •
453.6	3.985	0.6004	VII	◈ ▣ ▣
574.9	3.424	0.5345	VIIIa	⊥ I•I ⊥
706.3	2.951	0.4700	VI	ф ф ф
933.0	2.437	0.3869	I	◎ ◎ ◎
1283.9	1.887	0.2758	V	✥ ✥ ✥

These curves teach first the validity of the proposition that all energy curves represented logarithmically are congruent, and second, that law III holds good, as the observed points very closely follow its curve indicated by the dotted line. Consequently law IV appears to be confirmed in all its parts by the observations. A more rigorous test of the validity of the law can be obtained as follows:

If we write law IV in the form

IVa. $$\log (J \cdot \lambda^5) = \log c_1 - c_2 \cdot \log e \frac{1}{\lambda \cdot T},$$

the test of law IV may be reduced to that of a straight line, if we lay off $\log (J \cdot \lambda^5)$ and consider it as a function of $\frac{1}{\lambda T}$. The form of an energy curve $\left(\frac{1}{T} = \text{constant}\right)$ is hereby reduced to that of a straight line in the same way as in the case of an isochromatic line $\left(\frac{1}{\lambda} = \text{constant}\right)$ for which I had previously adopted this mode of representation. I gave in Table X the observed values of $\log (J\lambda^5)$ for a series of check measurements[1] which were made (1) with the reflecting hemisphere (copper oxide, Table VIIIb; or platinum black, Table VIIIa; designated in Table X by *CuO* or *Pt*); (2) with the platinum crucible cavity, which was covered with iron oxide[2] Table VI;

[1] These check measures were mostly made in connection with the measurements of the complete energy curves, the results of which are given in the proper tables. All the measures were made at exactly the same wave-lengths.

[2] All of the intensities J observed with this arrangement have been multiplied by the factor 2.421, in order that they may be comparable with the intensities with the other arrangements. The factor is the ratio of the values of J_m/T^5 in Tables VI and VIII.

TABLE X.

	Abs. T	$1/T$ 0.00 \ λ (μ) / $1/\lambda$	0.9104	1.208	1.589	1.971	2.316	3.247[1]	3.910	4.588	5.124[2]	6.263	7.736	8.244	Energy curves C_1	Energy curves C_2
			1.0983	0.8278	0.6294	0.5073	0.4318	0.3080	0.25574	0.21797	0.19517	0.15967	0.12927	0.12131		
H	1532.4	06526	0.618 −25	1.742 −14	2.562 −11	3.060 −15	3.373 −12	3.891 −3	4.115 +4	4.280 +15	4.374 +14	4.524 +17	4.663 +31	. . .	150300	14600
H	1292.7	07735	0.814−1 +11	1.098 −26	2.078 −14	2.669 −18	3.035 −21	3.641 −20	3.898 −18	4.089 −12	4.203 −8	4.379 −6	4.542 +8	. . .	142200	14520
H	1268.6	07883	0.718−1 +17	1.064 +18	2.050 +17	2.649 +7	3.025 +8	3.642 +10	3.904 +11	4.098 +17	4.214 +20	4.377 +6	4.542 +20	. . .	147900	14480
CuO	1134	08818	0.062−1 +10	0.561 +4	1.646 −16	2.330 −11	2.756 −5	3.449 −3	3.742 +1	3.960 +8	4.088 +15	4.267 −9	4.473 +28	4.456 +27	140900	14420
CuO	1121	08921	0.042−1 +60	0.499 −15	1.614 −8	2.304 −4	2.729 −4	3.431 0	3.727 +2	4.944 +7	4.072 +6	4.276 +10	4.470 +33	4.511 +29	144400	14510
CuO	1010.8	09893	. . .	0.986−1 −11	1.232 −3	1.992 −6	2.472 +4	3.250 +9	3.580 +12	3.820 +16	3.960 +13	4.186 +18	4.373 +15	4.417 +10	148100	14540
H	974.4	10263	. . .	0.835−1 +34	1.101 +16	1.876 −2	2.362 −6	3.158 −11	3.493 −15	3.740 −14	3.884 −16	4.119 −11	4.326 −2	. . .	148900	14540
CuO	929.9	10754	. . .	0.546−1 −1	0.894 0	1.718 −4	2.236 +2	3.078 +4	3.434 +5	3.695 +9	3.847 +7	4.093 +12	4.302 +14	4.352 +11	147600	14530
CuO	916.1	10916	. . .	0.469−1 +7	0.842 +13	1.664 −5	2.190 0	3.043 0	3.409 +6	3.671 +9	3.825 +5	4.074 +10	4.278 +4	4.323 −5	148000	14540
Pt	906.5	11031	. . .	0.407−1 +4	0.783 −1	1.626 −6	2.156 −2	3.023 +3	3.390 +6	3.655 +8	3.810 +4	4.063 +10	4.280 +15	4.347 +27	148900	14570
Pt	847.9	11704	. . .	0.011−1 +8	0.482 +2	1.386 −2	1.947 −4	2.874 +2	3.264 +3	3.545 +3	3.712 0	3.977 0	4.199 −3	4.255 −6	146100	14510
CuO	833.5	11997	. . .	0.872−2 −24	0.395 −3	1.307 −16	1.890 −5	2.830 −2	3.227 −1	3.514 0	3.683 −4	3.955 −1	4.179 −6	4.239 −7	145900	14530
Pt	830.3	12044	. . .	0.892−2 +19	0.373 −8	1.294 −14	1.873 −9	2.824 0	3.223 +2	3.510 +2	3.681 0	3.950 −1	4.186 +4	4.234 −9	147700	14560
Pt	822.1	12164	. . .	0.849−2 +38	0.336 +3	1.265 −5	1.845 −5	2.799 0	3.204 +3	3.497 +6	3.664 −2	3.936 −2	4.174 +2	4.220 −12	145400	14520
Pt	692.1	14449	. . .	. . .	0.453−1 +26	0.558 +19	1.231 +4	2.363 +6	2.836 +3	3.176 −1	3.383 −2	3.708 0	3.971 −14	4.050 −8	143200	14450
Pt	673.8	14841	. . .	. . .	0.283−1 +14	0.414 +1	1.118 −1	2.282 +2	2.771 +2	3.116 −7	3.332 −5	3.647 −22	3.943 −10	4.009 +15	142200	14460
Pt	660.0	15152	. . .	. . .	. . .	0.334 +21	1.046 +11	2.223 +3	2.717 −1	3.073 −7	3.287 −11	3.629 −8	3.908 −20	[illegible] −14	142200	14450
Isochromatic curves C_1			103200	138100	141100	141200	141500	144700	148500	147700	148900	145600	154800	146300		
Isochromatic curves C_2			14100	14460	14470	14480	14500	14500	14580	14550	14580	14510	14860	14660		

Mean values

	Formula IVa	Energy curves	Isochromatic curves
C_1	146030	145870	145300
C_2	14531	14514	14520

[1] At this wave-length an absorption averaging 2 per cent. is corrected by a constant factor 1.020.

[2] All the intensities J observed with this arrangement have been multiplied by 2.421 to render them comparable with the intensities with the other arrangement. This factor is the ratio of the values $J_m/T5$ in Table VI and Table VIII.

designated in Table X by large H). The numbers below give the difference, in units of the third decimal, of the observed values, and computed according to formula IV a (O.–C.) The following values of the constants were employed in the computation:

$$c_1 = 146030$$
$$c_2 = 14531$$

The numbers of the horizontal row represent an energy curve, which calculated as such, yield the constants printed at the right. The numbers in the vertical column represent an isochromatic curve, which calculated as such yields the constants printed below. We can, of course, locate each individual series within small limits of error on the corresponding straight line in the same manner as we may all of the numbers of all of the series of the comprehensive formula. But even the differences outstanding in this case correspond for the most part to only a few per cent. in the value of $J\lambda^5$. A deviation of 4, 9, 13, 17, 21, 25, etc., units of the third decimal correspond to 1, 2, 3, 4, 5, 6, etc., per cent. in the value of $J\lambda^5$. Within these limits of error the law is in any case demonstrated by this calculation.

TABLE XI

Abs. T.	λ (μ) / 1/λ ; 1/T 0.00	7.738 0.12924	6.263 0.15967	4.663 0.21444	3.354 0.2982	2.279 0.4389	1.887 0.5300	Energy curves C_1	C_2
723°.0[1]	1383	4.684	4.422	3.939	3.207	1.993	1.206	634600	14450
		+4	+6	−1	−7	−1	0		
577.1	1733	4.394	4.066	3.468	2.559	1.021	0.047	636700	14465
		−2	+1	−2	0	−7	+9		
462.4	2162	4.042	3.631	2.892	1.758	0.847−1		634100	14445
		−5	−3	+1	+4	+4			
373.0	2681	3.627	3.116	2.190	0.783			633100	14445
		0	+1	−3	−1				
Isochromatic curves[2] C_1		629600	642900	630300	623000	633700	600000[3]		
C_2		14440	14490	14450	14430	14460	14370		

[1] The wave-lengths belonging to this temperature were: 7.740μ, 6.264, 4.664, 3.356, 2.2181, 1889μ, and are calculated with these values.

[2] The isochromatic curves are here calculated on the basis of equal weight for all observations, (as in the computation of Formula IVa). Different weights were assigned in the earlier calculations.

[3] By an oversight the value 738800 was previously given at this point.

	Formula IVa	Mean values: Energy curves	Mean values: Isochromatic curves
C_1 - - -	633000	634600	631900
C_2 - - - -	14450	14450	14450

To facilitate comparison with the results at the lower temperatures I have calculated the set of observed numbers[1] previously communicated in the same manner, and give them in Table XI. We see that these observations yield a still closer validity of the law, as was to be expected from the theoretically better arrangement of the experiments.

Wien's law may accordingly be regarded as demonstrated as well as the difficulties of the experiments admit, within the range of wave-length from 9.2μ to 0.7μ (and in the research carried on in conjunction with Wanner,[2] to 0.5μ), and within the range of temperature from 1300° C. to 100° C.[3]

I consider the determination of the constant c_2 of the law as the final object of my researches. For lower temperatures I determined its value from experiments on cavities as $14455\mu \times$ degree of the absolute or of the Celsius scale.

From the radiation at lower temperatures produced by reflection Table VII gives

$c_2 = 5 \times \lambda_m \, . \, T = 5 \times 2890 = 14450$ with a mean error $= 15$.

The following value is yielded by the observations now communicated at higher temperatures. We have found as mean values of $\lambda_m \, . \, T$:

Table	Cavities: I	II	III	IV	V	VIa	VIb
$\lambda_m \, . \, T$ =	2932	2923	2915	2919	2910	2896	2901
Mean error =	4	8	5	8	6	4	5

Table	Reflection: VIIIa	VIIIb
$\lambda_m \, . \, T$ =	2899	2907
Mean error =	3.5	1

[1] Paper on lower temperatures. [2] This JOURNAL, 9, 300, 1899.

[3] Since in the observations of lower temperatures the radiation of the space in front of the slit contributes a considerable amount with the slide let down, which amount is added to the observed intensity, the lower temperature limit is the more correct.

This mean is

	$\lambda_m . T$	$c_2 = 5 \times \lambda_m . T$
with equal weights of all values -	2911	14555
with the omission of the values which seem least reliable in Tables I and II	2907	14535

The last value is in accord with that on which the calculation in Table X is based, *viz.*, $c_2 = 14531$.

The results of individual series deviating most from the means are, omitting Tables I and II:

Table	Temp. C.	$\lambda_m . T$	$c_2 - 5 \times \lambda_m . T$
VIa -	467.6	2862	14310
VIa -	- 1221	2941	14705

I estimate the highest possible error of the mean value of $\lambda_m . T$ to be 16; and hence that of c_2 to be 80.

A somewhat larger value is therefore obtained from the experiments at higher temperatures than from those at lower temperatures. We shall not be disposed to attach an excessive weight to the values of the constant c_2 found at higher temperatures when we consider the great experimental difficulties at high temperatures, and in particular the rather imperfect arrangement for producing the radiation, and further the still considerable uncertainties which seem to be possible in the determination of higher temperatures, according to the experiments of Holborn, Wien, and others. In my opinion the value $c_2 = 14455$ found with lower temperatures and relatively perfect arrangements is to be regarded as the more reliable,[1] similarly to the case of the determination of the constants of the law of total radiation for which a precise arrangement at low temperatures is preferred.

[1] I do not give a comparison of my results with those published by O. Lummer and E. Pringsheim (*Verhandlungen der Deutschen Physikalischen Gesellschaft*, 1, No. 1). as it was indicated in an address by Herr Pringsheim at the last meeting of the German Scientists' Association that the measurements are still being continued.