A GENERAL THEORY OF THE GLOW-LAMP.[1] I.

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THE phenomena of the glow-lamp are perhaps the simplest in the domain of radiation. The lamp consists of a conductor in a nearly complete vacuum, traversed by an electric current under the action of which it is heated to a stationary temperature such that the radiation from its surface consists in considerable part of visible rays. The simplicity of the conditions lead to the supposition that between the size and the temperature of the radiating surface, the wave-length and intensity of radiation, and the quantities by means of which the electrical energy developed in the filament are determined, there exist relations which are capable of expression in some simple form.

Up to the present time, only purely empirical attempts have been made to determine the relation between the amount of light emitted by the glow-lamp and its electrical constants. Jamieson and Preece believed the brightness of the glow-lamp to be proportional to the sixth power of the strength of the current. Voit concluded from the observations of Jamieson, and from numerous measurements made at the electrical exhibition of Munich upon many varieties of lamps, that the light was approximately proportional to the third power of the energy expended. Hess has shown that whenever the range of intensities becomes considerable, a member must be added to Voit's expression which is proportional to the energy. Finally, Slotte has drawn the conclusion from simultaneous measurements of the strength of the current and the intensity, that the light of the lamp is closely related to the fourth power of the current strength.

These experimental investigations could lead to no final result,

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since the most important element, viz. the temperature of the glowing filament, was left altogether out of account. Finding that the means of determining this temperature were lacking, only the most unreliable estimates concerning it, running from 2250° to 1200°, being attainable, I have spent several years in an attempt to establish an exact physical theory of the electrical glow-lamp which should cover all the observed facts, and have for this purpose made exact measurements upon more than thirty types of lamp, in order to determine the characteristic properties and to obtain the values of such constants of radiation for carbon as are necessary to a completion of the theory. I believe that I have now reached a point in these attempts where all the phenomena of the glow-lamp may be expressed by means of a few formulæ. Permit me to report briefly in the following pages the results of my investigations.

I.

The starting-point of my work was the establishment of a general expression by means of which the intensity of homogeneous radiation of a given wave-length emitted by a solid body at a given temperature might be expressed in terms of the wave-length and temperature, and of the size and nature of the radiating surface. No such general formula existed in the realm of radiation.

In 1888 I showed that the result of a great variety of observations could be expressed in this form, —

$$d^2s = \frac{dF \cdot dF_1 \cdot \cos w \cdot \cos w_1}{r^2} \cdot \frac{c}{\lambda^2} \cdot e^{aT - \frac{1}{b^2\lambda^2T^2}}, \qquad (1)$$

in which dF is an element of surface of the radiating body, dF_1 an element receiving this radiation, r the distance between the two elements of surface, w and w_1 the angles between the elements of these normals and the direction of the line joining them (r). This equation expresses the amount of energy of homogeneous radiation of the wave-length which is sent out from the surfaces of the element dF at the absolute temperature T to the surface of the element dF_1 in the unit of time. In this equation e has the usual signification; a is a constant common to all solids, viz. the value 0.0043; b^2 is a constant which should have the same, or nearly the same, value for various bodies; and, finally, c is a constant which varies from substance to substance. To distinguish them, I have named these three, respectively, constants of temperature (a), the illuminating power (b^2), and the constant of emissivity (c).

From Equation 1 it follows, in the first place, that the amount of energy radiated from the entire surface F in all directions in the unit of time is

$$s = c \cdot \pi \cdot F \cdot e^{aT - \frac{1}{b^2 \lambda^2 T^2}}. \tag{2}$$

This quantity is to be called the intensity of homogeneous radiation.

It follows, further, that the sum of the intensity of all the homogeneous rays which are emitted at the temperature T (the total radiation of the body) is expressed by the equation

$$S = \int_{\lambda=0}^{\lambda=\infty} s \cdot d\lambda = c \cdot \pi \cdot F \cdot \frac{\sqrt{\pi}}{2} \cdot b \cdot T \cdot e^{aT} = C \cdot F \cdot T \cdot e^{aT}. \tag{3}$$

The quantity $C = \frac{1}{2}\pi\sqrt{\pi c \cdot b}$ is to be called the constant of total radiation.

If the radiating body k is placed in an enclosed space, the walls of which have the common temperature T_0, the total loss of energy which K suffers in consequence of the radiation in the unit of time is given by the formula

$$\Delta S_{T,T_0} = C \cdot F(T \cdot e^{aT} - T \cdot e^{aT_0}), \tag{4}$$

it being assumed that the surface F of the radiating body is very small as compared with the surfaces of the surrounding walls, an assumption which is admissible for all incandescent lamps. I have assured myself that Equation 2 expresses the result of all trustworthy measurements hitherto made upon the relation between wave-length and temperature of homogeneous rays. And it can be shown also that Equation 4 agrees completely with the carefully determined results of Schleiermacher upon the total radiation of platinum and the oxide of copper for temperatures between 273° and 1173°. Measurements of my own upon the total radiation of carbon between 300° and 700° were also capable of being expressed by Formula 4.

II.

Equation 4 affords a simple method for the determination of the temperature of the glow-lamp. After the temperature of the filament under the action of the current has become stationary, the amount of energy developed by the current in each unit of time plus the energy which the filament receives from the surrounding walls is equal to the energy which it gives out, so that for every such time-interval the equation

$$\Delta S_{T,\,T_0} = C \cdot F(T \cdot e^{aT} - T_0 \cdot e^{aT_0}) = i \cdot \Delta P$$

holds true, where i and ΔP are the current intensity which is regarded as constant, and the difference of potential between the terminals of the filament, respectively.

In the case here assumed, where $\Delta S_{T,\,T_0}$ is given in heat units, the equation takes the following form:

$$\Delta S_{T,\,T_0} = C \cdot F(T \cdot e^{aT} - T_0 \cdot e^{aT_0}) = \frac{i \cdot \Delta P}{J},$$

in which J is the mechanical equivalent of the calorie. It follows from this equation that by means of measurements of the quantities i, ΔP, T and F, the temperature of the filament can be determined, provided the constant C is known. This can be found by the estimation of the stationary temperatures which the filament attains under the action of known currents and potential differences. The value of these temperatures is given from the resistance of the filament, provided the dependence of resistance upon temperature has been determined by a series of experiments for the intervals of temperatures not given. The following table gives an example of the determination of the constant C, the total radiation of the filament of a specimen of the new Cruto lamp, the surface of which had been found by the length of the carbon and its cross-section at eleven points distributed along the filament at equal distances. This surface was found to be 0.647 sq. cm. The resulting value of $C=0.0000171$ refers to gram-calories, centimeters, and seconds.

NEW CRUTO LAMP.

$F = 0.647$ sq. cm.

T	W	T	W
290°.0	332.08 Ω	440°.4	318.09 Ω
340°.3	327.68	490°.0	313.03
390°.3	323.01	540°.5	307.88

T_0 in the beginning = 290°.0; at the end = 290°.1.

i	ΔP	W	E	$\Delta S_{T,T_0}$	T	$\frac{T}{T_0}e^{a(T-T_0)}-1$	C
0.006526 A.	2.1472 V.	329.02 Ω	0.01401 W.	0.003344 cal.	325°.0	0.3027	169.2×10^{-7}
0.010051	3.2729	225.62	0.03289	0.007850	362°.5	0.7096	171.0
0.13176	4.2449	322.17	0.05593	0.013349	398°.9	1.1971	172.4
0.16542	5.2655	318.31	0.08710	0.020740	438°.2	1.8577	171.0
0.19986	6.2863	314.53	0.12564	0.029983	475°.3	2.6399	173.9
0.23358	7.2637	310.97	0.16966	0.040451	510°.2	3.5315	175.4
0.26690	8.2022	307.31	0.21892	0.052248	546°.1	4.6677	171.4

$M = 171.8 \times 10^{-7}$.

Following this plan, I have in the course of years determined the constant of total radiation for thirty-three varieties of glow-lamp. The summary of the results obtained shows that these thirty-three filaments belong to two groups. Three carbons, that of the Edison lamp, the new Cruto lamp, and of the lamp of Woodhouse and Rawson, have a radiation coefficient of 0.0000169 and 0.0000174; while thirty filaments give for the radiation-constant values between 0.0000127 and 0.0000132. The three carbons first-named are black; the others show gray surfaces. The mean value of C for these two different varieties of filament stand in the relation of 100 to 75.5. This is almost exactly the same relation which Leslie, at the beginning of this century, found for lamp-black and graphite; viz. 100 to 75. It may, therefore, be asserted that incandescent lamp filaments with gray surfaces should be classified as graphitic carbon.

A knowledge of the constant C makes it possible to determine the temperature of the filament for every condition of incandescence when we know the size of its radiating surface. It is interesting to know what the temperature of the filament is when the incandescent lamp is in operation at normal brightness. The result is a surprising one; viz. that the normal temperature of all sorts of lamps is very nearly the same, and is included in an interval between $T = 1565°$ and $T = 1580°$ of the absolute scale. In the case of some lamps of very great brilliancy — that is to say, of lamps with thick filaments, which can be brought to higher temperatures without serious damage, and which can consequently be operated at higher economy — we find the normal temperature to be about 40° higher.

It is, further, of importance to know between what temperatures the filament changes when the candle-power of the lamp is varied through considerable range: from 2 candles to 30 candles for a 16 candle-power lamp, for example, or from 20 to 300 candles in the case of a 200 candle-power lamp. The result of measurements shows that the temperature variations which correspond to such changes in brightness amount to about 180°. In practice, accordingly, glow-lamp illumination covers a range extending from 1400° to 1600° for small lamps, and 1450° to 1650° for large lamps. If we confine ourselves in the study of incandescent lamps to this range of temperatures, we may write, in place of Equation 5, the simple form —

$$\Delta S_{T, T_0} = C \cdot F \cdot T \cdot e^{aT} = \frac{i \Delta P}{J}, \tag{6}$$

since in these cases the quantity $T_0 \cdot e^{aT}$ is very small as compared with the quantity $T \cdot e^{aT}$. This is shown by the following small table: —

T_0	$T_0 \cdot e^{aT_0}$	T	$T \cdot e^{aT}$
273 + 50°	1.295 × 10⁸	1400°	576 × 10⁸
+ 75°	1.554	1450°	740
+ 100°	1.854	1600°	1554
+ 125°	2.203	1650°	1989

In this simpler form, Equation 6 indicates in what degree the current must increase in order to bring about a given rise of temperature in the filament. To make this matter clearer, I have drawn up the following table: —

T	$T \cdot e^{aT}$	T	$T \cdot e^{aT}$	T	$T \cdot e^{aT}$
1400°	576 × 10⁸	1500°	948 × 10⁸	1600°	1554 × 10⁸
1410	606	1510	996	1610	1634
1420	637	1520	1047	1620	1717
1430	670	1530	1100	1630	1803
1440	704	1540	1156	1640	1894
1450	740	1550	1214	1650	1989
1460	777	1560	1276	1700	2541
1470	816	1570	1340	1800	4136
1480	858	1580	1408	1900	6712
1490	902	1590	1479	2000	10860

From these data, and from the expression

$$\frac{d(T \cdot e^{aT})}{dT} \cdot \frac{1}{T \cdot e^{aT}} = \frac{1}{T} ta,$$

it may be seen that for the entire range of temperature from 1400° to 1600°, the following simple law holds true: viz. that every increase in the energy of the current amounting to ½ per cent raises the stationary temperature of the incandescent lamp 1°.

To show in greater detail the changes which the filament suffers by change of current and potential difference, and the changes which the resistance of the filament and the amount of light emitted suffer through change of temperature, I introduce the four following tables, which may also serve to indicate the manner in which the investigation of thirty-three types of incandescent lamps was carried out. In the eight vertical columns of the tables are given respectively ten values: The strength of the current i, the difference of potential ΔP, the resistance W, the energy E, the candle-power H, the quantity $\frac{H}{E_8}$, the efficiency $E_1 = \frac{E}{H}$, and the absolute temperature T. As unit of brightness, the British standard candle has been selected, and H denotes the mean horizontal intensity. Of the lamps to which these four tables refer, two had black carbons and two gray carbons. The quantity R in each table refers to the radiation factor for brightness—the factor, namely, by which the mean horizontal candle-power is to be multiplied in order to get the mean spherical candle-power. These quantities were determined by measuring the lamp in 132 positions as evenly distributed through space as possible.

WOODHOUSE AND RAWSON'S LAMP.

("60 V., 30 C.P.")

$F = 0.889$ sq. cm. $R = 0.780$. $C = 0.0000169$.

i	ΔP	W	E	H	$q = \frac{H}{E_3}$	$E_1 = \frac{E}{H}$	T
0.8768 A.	41.41 V.	47.23 Ω.	36.31 W.	1.82 N.K.	38.1×10^{-6}	19.95 W.	1400°
0.9855	45.18	45.85	44.52	3.60	40.8	12.37	1441
1.0967	49.01	44.69	53.74	6.55	42.2	8.20	1479
1.2108	52.79	43.60	63.90	11.07	42.4	5.77	1514
1.3264	56.72	42.76	75.23	17.85	41.9	4.21	1548
1.3853	58.60	42.30	81.18	22.38	41.8	3.63	1561
1.4460	60.59	41.90	87.61	27.61	41.1	3.17	1578
1.5058	62.48	41.49	94.08	34.13	40.9	2.76	1592
1.5675	64.46	41.12	101.05	41.16	39.9	2.55	1606
1.6290	66.32	40.71	108.63	48.35	38.4	2.23	1620

SUNBEAM LAMP.

("80 V., 5 A., 200 C. P.")

$F = 5.04$ sq. cm. $R = 0.711$. $C = 0.0000131$.

i	ΔP	W	E	H	$q = \frac{H}{E_2}$	$E_1 = \frac{E}{H}$	T
4.064 A.	53.90 V.	13.26 Ω.	219.0 W.	22.72 N.K.	2.16×10^{-6}	9.64 W.	1463°
4.218	55.74	13.21	235.1	28.47	2.19	8.26	1478
4.442	58.49	13.17	259.8	38.77	2.21	6.70	1498
4.747	62.26	13.12	295.5	56.82	2.20	5.20	1524
5.056	66.07	13.07	334.0	81.23	2.18	4.11	1549
5.377	70.00	13.02	376.4	115.1	2.16	3.27	1573
5.689	73.82	12.98	420.0	156.9	2.12	2.68	1595
5.980	77.49	12.96	463.4	205.8	2.07	2.25	1615
6.282	81.29	12.94	510.7	263.7	1.98	1.94	1636
6.584	84.99	12.91	559.6	334.7	1.89	1.69	1654

LAMP OF THE ALGEMEINEN ELEKTRIZITÄTS-GESELSHAFT, BERLIN.

("100 V., 16 C.P.")

$F = 0.760$ sq. cm. $R = 0.801$. $C = 0.0000129$.

i	ΔP	W	E	H	$q = \frac{H}{E_2}$	$E_1 = \frac{E}{H}$	T
0.4212 A.	77.19 V.	183.3 Ω.	32.51 W.	2.99 C.P.	87.2×10^{-6}	10.87 W.	1464°
0.4431	80.89	182.5	35.85	4.13	89.3	8.67	1483
0.4668	84.80	181.7	39.58	5.60	90.3	7.07	1503
0.4903	88.83	181.2	43.55	7.41	89.7	5.88	1522
0.5136	92.87	180.8	47.70	9.71	89.5	4.91	1541
0.5360	96.71	180.4	51.84	12.42	89.2	4.18	1557
0.5588	100.60	180.0	56.21	15.76	88.7	3.57	1574
0.5823	104.58	179.6	60.90	19.70	87.2	3.09	1591
0.6057	108.60	179.3	65.78	24.25	85.2	2.71	1607
0.6295	112.57	178.8	70.85	29.41	82.7	2.41	1621

NEW CRUTO LAMP.

("100 V., 0.59 A., 16 C.P.")

$F = 0.632$ sq. cm. $R = 0.734$. $C = 0.0000174$.

i	ΔP	W	E	H	$q = \frac{H}{E_3}$	$E_1 = \frac{E}{H}$	T
0.3948 A.	78.53 V.	198.9 Ω.	31.00 W.	2.21 C.P.	74.2×10^{-6}	14.03 W.	1434°
0.4233	83.40	196.9	35.30	3.32	75.9	10.63	1460
0.4506	87.56	194.3	39.45	4.69	76.4	8.41	1482
0.4924	93.79	190.5	46.18	7.58	77.0	6.08	1514
0.5180	97.60	188.4	50.56	9.93	76.8	5.09	1532
0.5444	101.12	185.7	55.05	12.75	76.4	4.32	1550
0.5706	104.41	182.9	59.58	16.10	76.1	3.70	1566
0.5990	108.27	180.4	64.85	20.67	75.8	3.14	1582
0.6305	112.11	178.7	70.68	26.60	75.3	2.66	1600
0.6579	115.99	176.3	76.32	33.06	74.2	2.31	1618